Greater New Haven Mathematics League
How Answers Are To Be Written 2019

Mathematical Ideas/Rules whose understanding will be assumed for all competitions:

Use of a non-technical dictionary in book form only (no computer generated versions) will be allowed for participants to whom English is not a first language, if so desired. Use of a visual aide such as a magnifying glass is allowed for those with a visual impairment. The student must present these to the proctor for inspection prior to the beginning of each round. If any question arises as to the legitimacy of the volume or device, the site chairperson will have the final word. Any QWERTY, TI-89, TI-92, HP 48, HP 49 series of calculators or any type of calculator that performs symbolic algebra are not allowed. **Calculators are permitted on the team round only.**

1. The word "*simplify"* will mean perform all operations. (Never leave information in factored form) All fractions must use a horizontal dividing symbol (e.g. $\frac{3}{4}$ not $3/4$).
2. The word "*compute*" will always call for a numeric answer in simplest form. Thus final answers like $\frac{6}{4}, 5+2, 2^5$, and $2\sin 30^o$, for example, would not be satisfactory. In cases where there is a question as to what is "most simplified", alternate answers may be accepted (example: $\frac{3}{2}$ and $1\frac{1}{2}$ are both acceptable). The judges' decision is final.
3. When an answer is called for as an *ordered pair (a,b)*, it must be given in precisely that form, including the parentheses and the comma. The same applies for other choices of letters and for ordered n-tuples.
4. *The sides opposite vertices A, B, and C of triangle ABC* will be represented by the lower case letters *a*, *b*, and *c*, respectively. Depending on the context, *A* can represent the vertex, or the angle, or the measure of the angle, and *a* can represent the side or its length. A similar convention holds for other choices of letters representing a triangle. If a quadrilateral is named *ARML*, it is understood that the vertices *A*, *R*, *M*, and *L* occur in this order around the polygon (either clockwise or counter clockwise). The convention holds for other choices of letters and for the naming of polygons in general. When referring to polygons (including triangles), we are referring to non-degenerate ones.
5. The *Fibonacci sequence* is a sequence 1, 1, 2, 3, 5, 8, 13, ..., where each term is the sum of the two previous terms, more formally, $F_1 = F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$.
6. *The Greatest Integer Function*, symbolized by brackets, is defined as follows: If *n* is an integer and if $n \leq x < n+1$ then $[x] = n$. Since brackets are often used simply as parentheses are, any problem using brackets to represent the *Greatest Integer Function* will clearly say so.
7. Logs are base 10 unless otherwise indicated: the use of log x also implies that x is positive. In general, when bases are not indicated, numbers referred to are in base 10. If another base is being used, that base will usually be written as a subscript. Examples: $\log_4 64$ (which equals 3); or the number $102.13_4 = 18\frac{7}{16}$.
8. The letter *i* will always be used for $\sqrt{-1}$.
9. Some symbols of *Combinations and Permutations*: $\binom{n}{r} = {}_nC_r = \frac{n!}{(n-r)!(r)!}$; this is the number of combinations of *n* things taken *r* at a time. ${}_nP_r = \frac{n!}{(n-r)!}$; this is the number of permutations of *n* things taken *r* at a time. Note: 0! = 1.

10. The capital A that begins the expression Arcsin x, Arccos x and Arctan x calls for the principal values of these inverse trigonometric functions. This also applies to the notations $\text{Sin}^{-1}x$, $\text{Cos}^{-1}x$ and $\text{Tan}^{-1}x$. The ranges are as follows: $-\frac{\pi}{2} \le \text{Arc}\sin x \le \frac{\pi}{2}$, $0 \le \text{Arc}\cos x \le \pi$; $-\frac{\pi}{2} < \text{Arc}\tan x < \frac{\pi}{2}$. A lower case letter, such as $\sin^{-1}x$ calls for all values $0 \le x < 2\pi$.
11. *Lattice points* are points on a grid, both of whose coordinates are integers.
12. *Divisors (or factors)* of an integer refers to positive divisors only. *Proper divisors* of an integer are divisors that are less than the integer itself.
13. The designation *primes* refers to positive primes only.
14. Sometimes problems refer to the digits of a number, in that case, those digits are usually underlined. Examples: "Let N = 7 7 7 . . . 7 7, where the digit 7 occurs 100 times"; or "Find A and B if K = A 2 5 B and K is a multiple of 72." (The number K is not to be interpreted as the product of A, 2, 5, and B.
15. If a *diagram* is given with a problem, it is not necessarily drawn to scale.
16. It is often helpful to have a basic knowledge of elementary number theory (including modular arithmetic) and of analytic geometry (including the conic sections) for these contests.
17. The *greatest lower bound* of a set is the largest number which is less than or equal to all the elements of the set. Thus 2 is the greatest lower bound for both $\{x : 2 < x\}$ and $\{x : 2 \le x\}$. The *least upper bound* of a set is the smallest number which is greater than or equal to all elements of the set. The 3 is the least upper bound for both $\{x : x < 3\}$ and $\{x : x \le 3\}$.
18. All answers will be exact unless otherwise specified by the problem writer.
19. *Ratio form*: "A:B", "A to B", $\frac{A}{B}$. If A and B are rational, they must be relatively prime integers. If either A or B is a radical, the rules for simplifying radicals with fractions must be used. e.g. $\sqrt{3} : \sqrt{2}$ must be written as $\sqrt{6} : 2$ and $2 : \sqrt{3}$ must be written as $2\sqrt{3} : 3$.
20. Simplest form of radical expressions:
    a) leaving no radical in the denominator
    b) leaving no fraction within the radical
    c) simplifying integral radicands so no integral radicand has an integral factor the nth power of any integer other than 1. e.g. $\sqrt{8} = 2\sqrt{2}$.
    d) All radicands must be stated using the lowest possible index, $\sqrt[4]{9} = \sqrt{3}$.
21. When writing the solutions of equations, a comma between solutions will be accepted. When writing solutions to inequalities the math symbols $\cup$ and $\cap$ are to be used. Interval and set notation are also acceptable.
22. In complex numbers, $\frac{a+bi}{c}$ or $\frac{bi+a}{c}$ form is acceptable, provided that a, b, and c are relatively prime and $c \neq 1$. Additionally, if the answer to a question is 5, 5 + 0*i* is also acceptable.
23. Unless clearly specified, polynomials that are to be factored should be factored over the RATIONALS.
24. Answers MUST be written on the ANSWER LINE at the bottom of the question sheet.
25. Conic Sections: "Standard form" will always mean the form that gives the vertex of a parabola or center of a circle, ellipse, or hyperbola. "General form" will always mean the expanded polynomial form in two variables.

Standard Form: $(x-h)^2 + (y-k)^2 = r^2$, $\frac{(x-h)^2}{a^2} \pm \frac{(y-k)^2}{b^2} = 1$
$y = a(x-h)^2 + k$, $(x-h)^2 = \frac{1}{a}(y-k)$, etc.

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$

Much of the above information is adapted from the CSAML Rules document.